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An investigation on quasi-normal fuzzy subgroups acted through group

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Abstract
Within the context of group theory, this study investigates the idea of quasi-normal fuzzy subgroups and their interactions. By adding graded membership to the conventional idea of subgroups, quasi-normal fuzzy subgroups provide a more complex understanding of subgroup structures. The study is focused on the behaviour of these fuzzy subgroups under group actions, which are transformations of sets reflecting group symmetries. This paper clarifies the structure and behaviour of quasi-normal fuzzy subgroups by examining how their characteristics, orbits, and stabilisers are preserved under group actions. This study has ramifications for algebraic topology, computer science, cryptography, and physics, among other mathematical fields. It also provides insights into the modelling and analysis of complex systems. Research in this field is expected to reveal new relationships and uses as it advances, enhancing.

Keywords: Quasi-normal fuzzy, subgroups, fuzzy subgroups

Introduction

A basic branch of mathematics known as group theory examines the characteristics and structures of groups, which are sets having an operation that complies with a set of axioms. Subgroups, which are subsets of a group that also form groups under the same operation, are an important term in this subject. Fuzzy sets and their applications in a variety of mathematical fields have attracted a lot of attention lately. One particular kind of fuzzy subgroup that shows interesting behaviours when operated upon inside groups are quasi-normal fuzzy subgroups. The purpose of this article is to examine the idea of quasi-normal fuzzy subgroups and how they behave under group actions [1].

It is necessary to first understand fuzzy subgroups in order to grasp quasi-normal fuzzy subgroups. A fuzzy subgroup of a group G is an extension of the traditional definition of a subgroup, in which membership is graded instead of binary, with a range of 0 to 1. An extra requirement pertaining to the normality of fuzzy subgroups is introduced by quasi-normal fuzzy subgroups. If there is a normal fuzzy subgroup N such that $N < F \leq NG$, where NG represents the fuzzy normal closure of F, then a fuzzy subgroup F of a group G is said to be quasi-normal.

Group actions, which characterise the symmetries found in group structure, are a basic idea in group theory. In formal terms, a group action is a mapping from a group G's components to transformations of a set X that meet certain criteria. Studying group activities allows one to learn about the composition, characteristics, and relationships of groups with other mathematical objects.

Analysing these fuzzy subgroups' behaviour under group action is a crucial part of the study of quasi-normal fuzzy subgroups acted by group actions. Comprehending how group activities maintain the characteristics of quasi-normal fuzzy subgroups is one line of inquiry. Furthermore, information on the structure and behaviour of quasi-normal fuzzy subgroups may be obtained by examining their orbits and stabilisers under group actions [2].

There are applications in many branches of mathematics and beyond for the study of quasi-normal fuzzy subgroups and associated operations within group theory. For example, knowing how quasi-normal fuzzy subgroups behave under group actions may have consequences for algebraic topology, where group actions are important for studying topological spaces. Moreover, the knowledge acquired from this study might have uses in the domains of computer science, cryptography, and physics, where complex systems are modelled and analysed using group theory [2].

Fuzzy quasi-normal subgroup acted by a group

The definition that follows may be seen as a generalisation of the idea of a fuzzy normal subgroup [3].

Definition 1: A fuzzy subgroup A of (S, Δ) acted by a group (G, +) is called a fuzzy quasinormal if its level subgroups of S are quasinormal subgroups in S acted by G.

Theorem 2: Any fuzzy normal subgroup under (S, Δ) acted by a group (G, +) is fuzzy quasinormal of S acted by G.
Proof. Clear, since any normal subgroup is quasinormal.

Theorem 3: The converse of theorem (2) is not true.
For that, consider the group. $S= \langle x,y|x^p = y^{p^3} = e,\ xyx^{-1} = y^{1+p}\rangle$, where p is an odd prime and e is the identity element of S. It is known that is a quasinormal subgroup but not normal. Define a fuzzy subset A of S as follows [4].

$$A(y) = \begin{cases} 1, & y = e \\ t_1, & y \in < x >\backslash < e > \\ t_2, & otherwise, 0 \le t_2 < t_1 \le 1 \end{cases}$$

Clearly, A is a fuzzy subgroup acted by a group (G, +). Moreover, it is quasinormal as its level subgroups $S \supset <x> \supset <e>$ are quasinormal acted by G. But G acts on A which is not a fuzzy normal subgroup as is not normal.
Let A be a fuzzy subgroup of S acted by a group (G, +), and Im A = {to, t1., tn}.
We may write A in the following form.

$$A = \sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right), \lambda_{A-1} = 0, A_{ti} \in \mathcal{F}_A$$

The following lemma is got first.

Lemma 4

Let a group (G, +) act on A under (S, Δ), and H, K be two subgroups of S acted by G. Let ti∈ Im A. Then: :(i) For $0 \le t \le 1$, ${}^tA$ is a fuzzy subgroup of S acted by G. Moreover, when t ≠ 0, ${}^tA$ and A have the same level subgroups acted by G, where $({}^tA)(x) = t\,A(x)$ for all $x \in G$. [5]

(ii) $\lambda_{HK} = \lambda_H o \lambda_K$.

(iii) $\left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right) o \lambda_K = \lambda_{A_{t1}} o\ \lambda_K - \lambda_{A_{t1-1}} o \lambda_K$

(iv) $\left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right) oB = \lambda_{A_{t1}} o\ B - \lambda_{A_{t1-1}}\ o\ B$ for any fuzzy subgroup B of G.

(v) $\left(t_i \lambda_{A_{t1}}\right) o \lambda_k = t_i \left(\lambda_{A_{t1}} o\ \lambda_k\right) = \lambda_{A_{t1}} o (t_i \lambda_k)$.

(vi) $\left[t_i(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}) + t_{i-1}(\lambda_{A_{t1-1}} - \lambda_{A_{t1-2}})\right] o\lambda_k = \left[t_i(\lambda_{A_{t1}} - \lambda_{A_{t1-1}})\ o\ \lambda_k + t_{i-1}(\lambda_{A_{t1-1}} - \lambda_{A_{t1-2}})\right] o\lambda_k$

(vii) $\left\{\sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right)\right\} o\ \lambda_k = \sum_{i=0}^{n} \left\{t_i(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}) o \lambda_k\right\}$

(viii) $\left\{\sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right)\right\} o\ B = \sum_{i=0}^{n} \left\{t_i(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}) o\ B\right\}$

For any fuzzy subgroup B of S acted by G.
Proof. (i) It is obvious.
(ii) Let s ∈ S, and x ∈ G. Then $\lambda_{KH}(x * s) = 1 \Leftrightarrow x \in HK \Leftrightarrow x * s = x * (s_1 s_2)$, where s1 ∈ H, $s_2 \in K \Leftrightarrow \lambda_H o \lambda_K(x * (s_1 s_2)) = 1 \Leftrightarrow \lambda_H o \lambda_K(x * s) = 1$.
Also $\lambda_{HK}(x * s) = 0 \Leftrightarrow x \notin HK \Leftrightarrow x * s$ cannot be written as a product of $x * (s_1 s_2)$, $s_1 \in H$, $s_2 \in K \Leftrightarrow x * s = x * (s_1 s_2)$ ($s_1 \in H$ and $s_2 \notin K$)

or ($s_1 \notin H$ and $s_2 \in K$) or ($s_1 \notin H$ and $s_2 \notin K$) $\Leftrightarrow \lambda_H o \lambda_K(x * s) = 0$.

(iii) we have $\left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right) o \lambda_k (x * s) = \begin{cases} 1, & x * s = uv, u \in \lambda_{A_{t1}} \backslash \lambda_{A_{t1-1}}, v \in K \\ 0, & otherwise \end{cases}$

$$= \begin{cases} 1, & x * s = uv\ (u \in A_{t1}, v \in K)\ and\ (u \notin A_{ti-1}, v \in K) \\ 0, & otherwise \end{cases}$$

$$= \lambda_{A_{t1}} o\ \lambda_K (x * s) - \lambda_{A_{t1-1}} o\ \lambda_K (x * s)$$

$$= (\lambda_{A_{t1}} o\ \lambda_K (x * s) - \lambda_{A_{t1-1}} o\ \lambda_K (x * s))$$

For (iv) - (viii), argument similar are used to that in (iii). It is remarkable to note that Lemma 3 (iii) - (viii) are valid if there is an action from the right by the composition law o in.

(iv)" $B\ o\ \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right) B o \lambda_{A_{t1}} - B o \lambda_{A_{t1-1}}$.

Theorem 5: Let a group (G, +) act on K as a subgroup of (S, Δ). Then K is quasinormal acted by G if and only if λk is a fuzzy quasinormal subgroup of S acted by G. Moreover, K is quasinormal if and only if $\lambda_k o \lambda_H = \lambda_H o \lambda_k$ for any subgroup H of S acted by G.

Proof: It is obtained from lemma (4) (ii)) and the definition (1) of quasinormality of K.
Theorem 6. Let a group (G, +) act on K as a subgroup of (S, Δ_). Then K is quasinormal acted by G if and only if $\lambda_k o A = A o \lambda_k$ for any fuzzy subgroup A of S acted by G.
Proof: Let a group (G, +) act on K as a subgroup of (S, Δ_). Such that $\lambda k o A = A o \lambda k$ for any fuzzy subgroup A of S acted by G. Therefore $\lambda_k o A = A o \lambda_k$ for any subgroup H of S acted by G.
Thus it follows, by lemma (4) (ii), that $\lambda_{KH} = \lambda_{HK}$ and so $KH = HK$ whence K is quasinormal acted by G,

Thus $A o \lambda_k = \{\sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right)\} o \lambda_k\ t_i \in Im\ A, \lambda_{A-1} = 0$

$$= \left\{\sum_{i=0}^{n} \left(t_i \lambda_{A_{t1}} - t_i \lambda_{A_{t1-1}}\right)\right\} o \lambda_k$$

$$= \sum_{i=0}^{n} \{ \left(t_i \lambda_{A_{t1}}\right) o \lambda_k - \left(t_i \lambda_{A_{t1-1}}\right) o \lambda_k$$

$$= \sum_{i=0}^{n} \{ t_i (\lambda_K o \lambda_k) - t_i (\lambda_K o \lambda_{A_{t1-1}})$$

$$= \lambda_K o \left\{\sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right)\right\}$$

$$= \lambda_K o A$$

A relation p is defined in $\mathcal{F}$ (the set of all fuzzy subgroups of S acted by G) as A p A' if and only if for all s, t in S, and x in G, A(x ∗ s) ≥ A(x ∗ t) ⇔ A'(x ∗ s) ≥A'(x ∗ t). This relation is an 136 equivalence relation in $\mathcal{F}$. Moreover, it is showed that if A, A" are any two fuzzy subgroups of S acted by G, then A p A" ⇔ $\mathcal{F}_A = \mathcal{F}_{A'}$. Thus without loss of generality, assume that there exists only one fuzzy subgroup A for each chain of subgroups of S as a level subgroups of A acted by G.

Composition properties on quasinormal fuzzy subgroup
Theorem 1: Let a group (G, +) act on a fuzzy subgroup A under (S, Δ). Then A is quasinormal acted by G if and only if A o B = B o A for any fuzzy subgroup B of S acted by G [6].

Proof. Let A be a fuzzy quasinormal subgroup of (S, Δ) acted by a group (G, +). Then $At1$ are quasinormal subgroups of S acted by G. Then $At1 \in \mathscr{F}A$, and so $\lambda At1$, are fuzzy quasinormal acted by G where 0≤ i ≤n. Therefore $\lambda_{A_{t1}} o\, B = B\, o\, \lambda_{A_{t1}}$, for any fuzzy subgroup B of S acted by G. Thus it gets that [7].

$$AoB = \left\{\sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right)\right\} oB$$

$$= \sum_{i=0}^{n} \{ t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right) oB\}$$

$$= \sum_{i=0}^{n} \{ t_i \left(\lambda_{A_{t1}} oB\right) - t_i (\lambda_{A_{t1-1}} oB)$$

$$= \sum_{i=0}^{n} \{ t_i \left(Bo\lambda_{A_{t1}}\right) - t_i (Bo\lambda_{A_{t1-1}})\}$$

$$= \sum_{i=0}^{n} \{ Bot_i (\lambda_{A_{t1}} - \lambda_{A_{t1-1}})\}$$

$$= Bo\left\{\sum_{i=0}^{n} t_i \left(\lambda_{A_{t1}} - \lambda_{A_{t1-1}}\right)\right\}$$

$$= BoA$$

To prove the converse, assume that A is a fuzzy subgroup of S acted by G such that A o B = B o A for any fuzzy subgroup B of S acted by G. Thus $Ao\lambda_K = \lambda_K oA$ for any subgroup K of S acted by G. Now a fuzzy subgroup A" of S is defined. For 0<s<1, Im $A'' = \{s^{\circ}, s^{1}, s^{2} \ldots\ldots s^{n}\}$ where $A_{s^0} = A_{t0}, A_s = A_{t1}, A_{s^2} = A_{t1}, \ldots, A_{s^n} = A_{tn} = G$. Thus A and A" have the same chain of level subgroups acted G. Hence if A" is quasinormal of S acted by G, then A does by definition [8].

Now it finds that

$$A' = \textstyle\sum_{i=0}^{n} s^i \left(\lambda_{As^i} - \lambda_{As^{i-1}}\right), \text{ where } \lambda_{As^{-i}} = 0.$$

Hence for all u in S, and x in G, $Ao\lambda_k(x * s) = \lambda_k oA(x * s)$.

$$\Rightarrow \{\textstyle\sum_{i=0}^{n} s^i \left(\lambda_{As^i} - \lambda_{As^{i-1}}\right)\} o\lambda_k (x * u) = \lambda_k o\{\sum_{i=0}^{n} s^i \left(\lambda_{As^i} - \lambda_{As^{i-1}}\right)\}(x * u).$$

$$\Rightarrow \textstyle\sum_{i=0}^{n} s^i \left\{\left(\left(\lambda_{As^i} o\lambda_k - \lambda_k o\lambda_{As^i}\right) - \left(\lambda_{As^i} o\lambda_k - \lambda_k o\lambda_{As^{i-1}}\right)\right)\right\}(x * u) = 0.$$

$$\Rightarrow \left\{\left(\left(\lambda_{As^i} o\lambda_k - \lambda_k o\lambda_{As^i}\right) - \left(\lambda_{As^{i-1}} o\lambda_k - \lambda_k o\lambda_{As^{i-1}}\right)\right)\right\}(x * u) = 0.$$

$$\Rightarrow \left(\lambda_{As^i} o\lambda_k - \lambda_k o\lambda_{As^i}\right)(x) = \left(\lambda_{As^{i-1}} o\lambda_k - \lambda_k o\lambda_{As^{i-1}}\right)(x * u) \text{ for all } u \in S, x \text{ in } G, \text{ and } 0 \le i \le n$$

$$\Rightarrow \left(\lambda_{As^i} o\lambda_k - \lambda_k o\lambda_{As^i}\right)(x) = 0$$

(because if not, then there exists u∈ S such that $(\lambda_{As^i} o\lambda_k - \lambda_k o\lambda_{As^i})(x * u)$=1 or -1 (say 1).

Then $\lambda_{As^i} o \lambda_k(x * u) = 1$ and $\lambda_k o \lambda_{As^i}(x * u)$=0 for all 0≤i≤n.

Thus $\lambda_G o \lambda_k(x * u) = 1$ and $\lambda_K o \lambda_G(x * u) = 0)$ and have $\lambda_{GK}(x * u) = 1$, $\lambda_{KG}(x * u) = 0$.

Therefore u ∈ GK and u ∉ KG which is a contradiction). Then it follows that [9].

$$\lambda_{As^i} o \lambda_k - \lambda_k o \lambda_{As^i} \text{ and so } \lambda_{As^i K} = \lambda_{KAs^i} \text{ for all } 0 \leq i \leq n.$$

Thus $As\ iK = KAs\ i$ for any subgroup K of S acted by G, 0 ≤ i ≤ n, and so $As\ i$ is quasinormal subgroup of S acted by G for all 0≤i≤n. Hence $At1$ is a quasinormal subgroup of s acted by G for all 0≤i≤n, and therefore A is fuzzy quasinormal of S acted by G. The operation o is associative. Also if A and B are two fuzzy subgroups of G, then A o B is a fuzzy subgroup of G if and only if A o B = B o A. As a consequence result of this result, we have the following two corollaries [10].

Corollary 2
If A is a fuzzy quasinormal subgroup acted by a group (G, +), and B is any fuzzy sub#roup of (S, Δ) acted by G, then A o B is a fuzzy subgroup of S acted by G.

Corollary 3
If a group (G, +) acts on two fuzzy quasinormal subgroups of (S, Δ), then A o B is fuzzy quasinormal subgroup of S acted by G.
Proof. From Corollary (2), it follows that A o B is a fuzzy subgroup of S acted by G. To show A o B is fuzzy quasinormal acted by G, let C be any fuzzy subgroup of S acted by G. Then.

$$\begin{aligned}(AoB)oC &= Ao(BoC) \text{ (}o\text{ is associative)}\\ &= Ao(CoB) \text{ (B is quasinormal)}\\ &= (AoC)oB \text{ (}o\text{ is associative)}\\ &= (CoA)oB \text{ (A is quasinormal)}\\ &= Co(AoB) \text{ (}o\text{ is associative)}\end{aligned}$$

Thus A o B is fuzzy quasinormal acted by G from theorem (1).
The previous corollary informs us that Q(SG), the set of all fuzzy quasinormal subgroups of S acted by a group (G, +). is closed under operation o and since o is an associative, it follows that (Q(G), o) is a commutative semigroup. Also since for any fuzzy subgroup A of G, A o A = A. Then we have the following result.

Corollary 4
(Q(G), o) is a commutative idempotent semigroup.

Definition 5
Let S be a semigroup and m be a non-empty subset of S
(i) m is called a left (right) ideal of S if $m \supset S, m (m \supset mS)$ and it is an ideal if it is a left and a fight ideal; (ii) S is called semisimple if $m2 = m$ for any ideal m of S.

Definition 6
Let $\mathcal{A}$ and $\mathcal{B}$ be two subsets of Q(SG). We define $\mathcal{A}$ o $\mathcal{B}$~ = {A o B | A∈ $\mathcal{A}$, B∈ $\mathcal{B}$} and $\mathcal{A}2 = \mathcal{A}o\mathcal{A}$

Theorem 7
(Q(SG), o) is a commutative idempotent semisimple semigroup.
Proof. From Corollary (7.4.2), it needs only to show that (Q(SG), o) is semisimple. Let ~¢ be an ideal in Q(SG). Then $\mathcal{A} \supset \mathcal{A}^2$. Further, if A∈ $\mathcal{A}$, then A = A o A∈ $\mathcal{A}^2$ and so $\mathcal{A}^2 \supset \mathcal{A}$.. Thus $\mathcal{A}2 = \mathcal{A}$ for any ideal $\mathcal{A}$ in Q(G) and hence (Q(SG), o), is semisimple semigroup.
Theorem (7) may be considered as a generalization of the result. Moreover ((G), o) is a subsemigroup of (Q(SG), o), where $\mathcal{N}$(SG) is the set of all fuzzy normal subgroups of G.

Definition 8
A fuzzy subgroup of (S, Δ) acted by a group (G, +) is called subnormal acted by G if its level subgroups are subnormal acted by G.

Theorem 9
Any fuzzy quasinormal subgroup of S acted by G is a fuzzy subnormal acted by G.
Proof. The proof is obvious by using theorem (9) and the definitions of fuzzy quasinormality and subnormality acted by G.
The converse of theorem (9) need not be true.
For that, consider G = A¢ (alternating group of 4-letters).
Define a fuzzy subset A as follows.

$$A(x) = \begin{cases} 1, & x = 1 \\ t_1, & x = (12)(34), \\ t_2, & x \in N\langle(1,2)(3,4)\rangle \\ 0, & otherwise, \ 1 \geq t_1 > t_2 \geq 0 \end{cases}$$

where N is the Klein four group: $N = \{(1),(12)(34),(13)(24),(23)(14)\}$.
Clearly, A is a fuzzy subgroup as its level subsets are subgroups acted by a group (, +). Now the level subgroups of A form a normal chain: ((1)) ⊲ ((12) (34)), ⊲N⊲A4 = G. Thus A is fuzzy subnormal which is not quasinormal acted by G, as ((12) (34)) is a level subgroup of A but is not quasinormal acted by G.

Corollary 10
Let G be a group of square free order and A be a fuzzy subgroup of S acted by G. Then A is quasinormal acted by g if and only if A is normal acted by G.
Proof. Let A be fuzzy quasinormal of S acted by G. Then its level subgroups are quasinormal subgroups in S acted by G. Thus by (9), the level subgroups of A are subnormal acted by G. But as S is square free order, then any level subgroup $At1(t1 \in ImA)$ is a Hall subgroup of S actedby G. Thus At1, is normal acted by G for all $ti \in$ Im A, and hence A is fuzzy normal acted by G. The converse is (2) [11].

Conclusion
Quasi-normal fuzzy subgroups offer a fascinating avenue for exploration within group theory, particularly when studied in the context of group actions. Investigating how these fuzzy subgroups behave under group actions can deepen our understanding of their structure and properties, with potential applications in various mathematical and interdisciplinary fields. As research in this area progresses, it is likely to uncover new connections and insights, further enriching our understanding of group theory and its applications.

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